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Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2017
First Degree Programme under CBCSS
MATHEMATICS
Core Course XII
MM 1644 : Graph Theory
(2013 Admn.)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry 1 mark **each**.

1. Find the number of edges in a complete graph on 1000 vertices.
2. How many edges are there in a spanning tree for a simple connected graph with 24 vertices ?
3. Give an example of a pseudosymmetric digraph.
4. Show by example that a radius in a tree is not necessarily half its diameter.
5. Draw a complete bipartite graph with an Euler circuit.
6. Let G be a simple graph with n vertices and k connected components. What is the minimum possible number of edges of G ?
7. Which is the nonplanar graph with the smallest number of vertices ?
8. Draw a graph whose incidence matrix is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
9. Give the adjacency matrix of a complete graph on 4 vertices.
10. Identify the class of connected graphs G such that G – e is disconnected for each edge e of G.

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SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Let G be a regular graph of vertex degree k, an odd number. Prove that the number of edges in G is a multiple of k.

12. Draw two non-isomorphic graphs with the same number of vertices, same number of edges and an equal number of vertices with a given degree.

13. If a graph has exactly two vertices of odd degree show that there must be a path joining these two vertices.

14. Explain the Konigsberg bridge problem and draw the graph corresponding to it.

15. If a simple graph G has at least two vertices, show that it has two vertices of the same degree.

16. Show that every connected graph has at least one spanning tree.

17. Prove that no simple graph with two or three vertices is self-complementary.

18. Show that a bipartite graph contains no circuit of odd length.

19. Prove or disprove : Every Euler simple graph with an even number of vertices has an even number of edges.

20. Show that any tree with two or more vertices has at least two pendant vertices.

21. In any digraph G, prove that $\sum_{i=1}^{n} d^{+}(v_i) = \sum_{i=1}^{n} d^{-}(v_i)$, each sum being equal to the number of edges in G.

22. If a graph G on n vertices has fewer than $3n - 6$ edges, must G be planar ? Justify your answer.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Show that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.

24. If A(G) is an incidence matrix of a connected graph G with n vertices, show that the rank of A(G) is $n - 1$.

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25. Show that a tree with n vertices has n – 1 edges.

26. In a class with 9 students, each student sends a card to 3 other students. Is it possible that each student receives cards from the same 3 students to whom he/she sent cards ? Justify your answer.

27. Let G be a graph with exactly two vertices x and y of odd degree. Let G^* be the graph obtained by putting a new edge (x, y) joining x and y. Prove that G is connected if and only if G^* is connected.

28. Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

29. Show that a graph G is a tree if and only if there is a unique path between any two vertices of G.

30. Let G be a simple connected graph with at least 11 vertices. Prove that either G or its complement $\overline{G}$ must be nonplanar.

31. Given a simple graph and its complement, prove that either of them is always connected.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Explain the Teleprinter's problem. Illustrate the method of solution using Euler digraph with an example.

33. a) Show that every tree has either one or two adjacent centers.
    b) Show that the Kuratowski's graphs are nonplanar.

34. a) If G is a simple connected planar graph with n vertices and e edges, show that $e \leq 3n - 6$.
    b) Show that the maximum number of edges in a complete bipartite graph with n vertices is $[n^2/4]$.

35. a) Let G be a graph with exactly 10 vertices and 27 edges. Suppose that each vertex has degree 3, 5 or 7 and that there are exactly 2 vertices of degree 5. How many vertices of degree 7 does G have ? Justify your answer.
    b) Show that a graph G with n vertices, n – 1 edges and no circuits is connected.

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